Water-Temperature-Dependent Wet Bulb Temperature Calculation

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Abstract

A numerical solution for the exact calculation of the wet bulb temperature of air has been derived, implemented in Visual Basic for Applications, and validated against the Air Humid Handling reference software. The calculation method is based on the thermodynamic wet bulb temperature calculation, described in the 2009 ASHRAE Handbook-Fundamentals, but has been adapted by introducing water-temperature-dependent behavior.

1 Introduction

The wet bulb temperature is the temperature associated with the state in which, during an adiabatic process, water vapor reaches its saturation pressure. As air exchanges sensible enthalpy for latent enthalpy, the net change in enthalpy is determined exclusively by the enthalpy of the evaporated water.

The definition of the thermodynamic wet bulb temperature implies the assumption that liquid water that is being evaporated has a dry bulb temperature equal to the resulting wet bulb temperature of the air it is adiabatically added to.

In practice, however, the added water usually has a different temperature, leading to a small but measurable difference in the resulting wet bulb temperature.

This paper presents a method of calculating the realistic wet bulb temperature using a minimum amount of measurable quantities, hereby minimising the experimental uncertainty. An implementation in VBA (Visual Basic for Applications) is also included, as well as a validation of the derived calculation method.

The equations representing the properties of moist air were obtained from the psychrometrics section of the 2009 ASHRAE Handbook-Fundamentals [1].

2 Calculation

2.1 Input parameters

The wet bulb temperature is preferably calculated using four measurable quantities: the dry bulb temperature, the dew point temperature, the water temperature, and the atmospheric pressure. This approach differs from the ASHRAE handbook, as it does not include the water temperature as a separate parameter affecting the wet bulb temperature.

An alternative method of determining the starting point of the adiabatic process is using the relative humidity instead of the dew point temperature, although, in practice, this may lead to a less accurate result.

2.2 Psychrometrics

The first step is to calculate the water vapor saturation pressure associated with the dew point (valid between 0 °C and 200 °C). Note that equation 1, 3 and 12 depend on the absolute temperature, denoted by the subscript k.

$$\ln p_{ws} = \frac{c_8}{T_{dp,k}} + c_9 + c_{10} T_{dp,k} + c_{11} T_{dp,k}^2 + c_{12} T_{dp,k}^3 + c_{13} \ln T_{dp,k} \tag{1}$$

where

$$\begin{aligned} c_8 &= -5.8002206 \times 10^3 \\ c_9 &= 1.3914993 \times 10^0 \\ c_{10} &= -4.8640239 \times 10^{-2} \\ c_{11} &= 4.1764768 \times 10^{-5} \\ c_{12} &= -1.4452093 \times 10^{-8} \\ c_{13} &= 6.5459673 \times 10^0 \end{aligned}$$

This can be used to calculate the absolute humidity at the dew point, which, by definition, is equal to that of the starting point, as they lie on the same isodrosotherm.

$$W \equiv W_{dp} = 0.621945 \frac{p_{ws}}{100p - p_{ws}} \tag{2}$$

Alternatively, if the initial relative humidity is known, the partial vapor pressure of unsaturated air can be calculcated,

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that can be used to determine the absolute humidity of the starting point. Again, note the use of the absolute temperature.

$$\ln\, p_{ws} = \frac{c_8}{T_k} + c_9 + c_{10} T_k + c_{11} T_k^2 + c_{12} T_k^3 + c_{13} \ln\, T_k \tag{3}$$

$$p_w = \frac{\phi}{100} p_{ws} \tag{4}$$

$$W = 0.621945 \frac{p_w}{100p - p_w} \tag{5}$$

The second step is to derive a set of equations containing the unknown wet bulb temperature and corresponding absolute humidity. Conservation of enthalpy requires that

$$h + \left(W_s^* - W\right) h_w = h_s^* \tag{6}$$

The specific enthalpy of unsaturated and saturated air is given by

$$h = 1.006T + W\left(2501 + 1.86T\right) \tag{7}$$

and

$$h_s^* = 1.006T^* + W_s^*\left(2501 + 1.86T^*\right) \tag{8}$$

respectively. The specific enthalpy of the added water (above 0 °C) is approximated by

$$h_w \approx 4.186 T_w \tag{9}$$

Equation 9 is fundamentally different from the ASHRAE handbook, that states that water is added at a temperature equal to the (yet to be found) wet bulb temperature. This method, however, more realistically introduces the water temperature as a measurable fixed value. The resulting absolute humidity (i.e. at the wet bulb) can be expressed by substituting equation 7-9 in equation 6.

$$W_s^* = \frac{W\left(2501 + 1.86T - 4.186T_w\right) + 1.006\left(T - T^*\right)}{2501 + 1.86T^* - 4.186T_w} \tag{10}$$

This unknown absolute humidity is also given by

$$W_s^* = 0.621945 \frac{p_{ws}^*}{100p - p_{ws}^*} \tag{11}$$

where, again expressed in the absolute temperature,

$$\ln\, p_{ws}^* = \frac{c_8}{T_k^*} + c_9 + c_{10} T_k^* + c_{11} T_k^{*2} + c_{12} T_k^{*3} + c_{13} \ln\, T_k^* \tag{12}$$

This leaves two equations having two unknown quantities. The third step is to solve this, by merging equation 10 and 11 into one, hereby eliminating the unknown absolute humidity. The result is a single equation containing the wet bulb temperature as the only unknown quantity. This is expressed by introducing a new function y.

$$\begin{gathered} y \equiv \left(100p - p_{ws}^*\right)\left\{W\left(2501 + 1.86T - 4.186T_w\right)\right. \\ \left. + 1.006\left(T - T^*\right)\right\} - 0.621945 p_{ws}^*\left(2501 + 1.86T^* - 4.186T_w\right) \end{gathered} \tag{13}$$

2.3 Numerical solution

Equation 13 can be solved numerically using the iterative Newton-Raphson method.

$$T_{n+1}^* = T_n^* - \frac{y_n}{y_n^{'}} \tag{14}$$

The initial value (T_0^*) is chosen to be equal to the water temperature. However, calculation of the derivative of function y is a rather complex and very elaborate task. A much simpler, but in practice equally accurate method, is using the definition of the derivative of a function.

$$y_n^{'} = \frac{dy_n}{dT^*} = \lim_{\triangle T^* \to 0} \frac{\triangle y_n}{\triangle T^*}$$

$$= \lim_{\triangle T^* \to 0} \frac{y\left(T_n^*\right) - y\left(T_n^* + \triangle T^*\right)}{\triangle T^*} \tag{15}$$

The derivative can be approximated by choosing a value for $\triangle T^*$ that is sufficiently small (e.g. 0.001 K). The iteration will continue until a chosen accuracy is met (e.g. 0.01 K), but will usually finish after only a few steps, as this is usually a quickly converging iteration method.

Although some programming languages have built-in modules for solving equation 13 directly, the advantage of deriving an iteration algorithm is that the result will be much less platform dependent.

3 VBA code

The numerical solution method has been implemented in VBA. It also contains the option of choosing between either the dew point temperature or the relative humidity as the secondary input parameter besides the dry bulb temperature. An unrealistic value of -999999 will be returned when the required accuracy cannot be reached within the chosen maximum number of iteration steps.

```
Function WB(InputType As String, T_in,
DP_or_RH, T_water, p As Single) As Single

Dim c8, c9, c10, c11, c12, c13, accuracy, max_steps,
TDP_in_K, pws_in, W_in, T_in_K, pw_in,
T_out_new, i, T_out_K, pws_out, y, T_out_step,
pws_out_step, y_step, delta_T_out As Single

’Declaration of constants
c8 = -5800.2206
c9 = 1.3914993
c10 = -0.048640239
c11 = 0.000041765768
c12 = -0.000000014452093
c13 = 6.5459673
accuracy = 0.01
max_steps = 10

’Check input type and calculate inlet absolute
humidity
If InputType = "DP" Then
   TDP_in_K = DP_or_RH + 273.15
   pws_in = Exp(c8 / TDP_in_K + c9
+ c10 * TDP_in_K + c11 * (TDP_in_K ^ 2)
+ c12 * (TDP_in_K ^ 3) + c13 * Log(TDP_in_K))
   W_in = 0.621945 * pws_in / (100 * p - pws_in)
ElseIf InputType = "RH" Then
   T_in_K = T_in + 273.15
   pws_in = Exp(c8 / T_in_K + c9
+ c10 * T_in_K + c11 * (T_in_K ^ 2)
+ c12 * (T_in_K ^ 3) + c13 * Log(T_in_K))
   pw_in = (DP_or_RH / 100) * pws_in
   W_in = 0.621945 * pw_in / (100 * p - pw_in)
Else
   WB = -999999
   Exit Function
End If

’Initial assumption
T_out_new = T_water

’Iteration
For i = 1 To max_steps
   T_out = T_out_new
   T_out_K = T_out + 273.15
   pws_out = Exp(c8 / T_out_K + c9
+ c10 * T_out_K + c11 * (T_out_K ^ 2)
+ c12 * (T_out_K ^ 3) + c13 * Log(T_out_K))
   y = (100 * p - pws_out) * ((2501 + 1.86 * T_in
- 4.186 * T_water) * W_in
+ 1.006 * (T_in - T_out)) - 0.621945 * pws_out
* (2501 + 1.86 * T_out - 4.186 * T_water)

   T_out_step = T_out + 0.001
   T_out_step_K = T_out_step + 273.15
   pws_out_step = Exp(c8 / T_out_step_K + c9
+ c10 * T_out_step_K
+ c11 * (T_out_step_K ^ 2)
+ c12 * (T_out_step_K ^ 3)
+ c13 * Log(T_out_step_K))
   y_step = (100 * p - pws_out_step)
* ((2501 + 1.86 * T_in - 4.186 * T_water)
* W_in + 1.006 * (T_in - T_out_step))
- 0.621945 * pws_out_step
* (2501 + 1.86 * T_out_step - 4.186 * T_water)

   T_out_new = T_out - y / ((y_step - y)
/ (T_out_step - T_out))
   delta_T_out = Abs(T_out_new - T_out)
   If delta_T_out < accuracy Then Exit For
Next i

’Output
If delta_T_out < accuracy Then
   WB = T_out_new
Else
   WB = -999999
End If

End Function
```

4 Validation

4.1 Reference software

The output of the VBA code is compared to the results obtained with the psychrometric calculation program Air Humid Handling (AHH) 7.1 [2]. The validation also includes the required number of iterations to achieve an accuracy of 0.01 K.

The dry bulb temperature, water temperature, and atmospheric pressure were varied independently, around the base input of 32.8 °C, 15 °C and 1013.25 hPa, respectively. The base value of the secondary input parameter is either 14.4 °C dew point temperature or 33% relative humidity.

These initial air conditions originate from the California Appliance Efficiency Regulations [3], requiring 91 °F (about 32.8 °C) dry bulb temperature and 69 °F (about 20.6 °C) wet bulb temperature for testing evaporative air coolers following procedures as defined in ANSI/ASHRAE Standard 133-2008 [4] and ANSI/ASHRAE Standard 143-2007 [5].

4.2 Dew point temperature as secondary input parameter

Table 1 – $T_{\mathrm{dp}} = 14.4$ °C, $T_{\mathrm{w}} = 15$ °C, $p = 1013.25$ hPa

T [°C]	T^* AHH [°C]	T^* VBA [°C]	Difference [K]	Iterations [-]
20	16.46	16.46	0.00	3
25	18.18	18.16	-0.02	3
32.8	20.64	20.62	-0.02	3
40	22.71	22.68	-0.03	4
50	25.32	25.28	-0.04	4

Table 2 – $T = 32.8$ °C, $T_{\mathrm{w}} = 15$ °C, $p = 1013.25$ hPa

T_{dp} [°C]	T^* AHH [°C]	T^* VBA [°C]	Difference [K]	Iterations [-]
0	15.41	15.39	-0.02	2
5	16.84	16.81	-0.03	3
14.4	20.64	20.62	-0.02	3
20	23.68	23.67	-0.01	4
25	26.91	26.90	-0.01	4

Table 3 – $T = 32.8$ °C, $T_{\mathrm{dp}} = 14.4$ °C, $p = 1013.25$ hPa

T_{w} [°C]	T^* AHH [°C]	T^* VBA [°C]	Difference [K]	Iterations [-]
5	20.58	20.55	-0.03	4
10	20.61	20.58	-0.03	4
15	20.64	20.62	-0.02	3
35	20.76	20.74	-0.02	4
60	20.93	20.91	-0.02	5

Table 4 – $T = 32.8$ °C, $T_{\mathrm{dp}} = 14.4$ °C, $T_{\mathrm{w}} = 15$ °C

p [hPa]	T^* AHH [°C]	T^* VBA [°C]	Difference [K]	Iterations [-]
800	19.78	19.76	-0.02	3
900	20.20	20.18	-0.02	3
1013.25	20.64	20.62	-0.02	3
1100	20.95	20.92	-0.03	3
1200	21.28	21.26	-0.02	3

4.3 Relative humidity as secondary input parameter

Table 5 – $\phi = 33\%$, $T_{\mathrm{w}} = 15$ °C, $p = 1013.25$ hPa

T [°C]	T^* AHH [°C]	T^* VBA [°C]	Difference [K]	Iterations [-]
10	3.92	3.93	0.01	4
20	11.33	11.33	0.00	3
32.8	20.62	20.63	0.01	3
40	25.88	25.88	0.00	4
50	33.31	33.30	-0.01	5

Table 6 – $T = 32.8$ °C, $T_{\mathrm{w}} = 15$ °C, $p = 1013.25$ hPa

ϕ [%]	T^* AHH [°C]	T^* VBA [°C]	Difference [K]	Iterations [-]
10	14.76	14.75	-0.01	2
20	17.46	17.46	0.00	3
33	20.62	20.63	0.01	3
50	24.28	24.28	0.00	4
65	27.14	27.14	0.00	4

Table 7 – $T = 32.8$ °C, $\phi = 33\%$, $p = 1013.25$ hPa

T_{w} [°C]	T^* AHH [°C]	T^* VBA [°C]	Difference [K]	Iterations [-]
5	20.56	20.56	0.00	4
10	20.59	20.59	0.00	4
15	20.62	20.63	0.01	3
35	20.75	20.75	0.00	4
60	20.92	20.92	0.00	5

Table 8 – $T = 32.8$ °C, $\phi = 33\%$, $T_{\mathrm{w}} = 15$ °C

p [hPa]	T^* AHH [°C]	T^* VBA [°C]	Difference [K]	Iterations [-]
800	19.77	19.77	0.00	3
900	20.19	20.19	0.00	3
1013.25	20.62	20.63	0.01	3
1100	20.93	20.93	0.00	3
1200	21.27	21.27	0.00	3

5 Conclusions

- A relatively simple and efficient method of calculating the wet bulb temperature has been developed based on the 2009 ASHRAE Handbook-Fundamentals, and was successfully implemented in VBA.
- The required input parameters are limited to the dry bulb temperature, water temperature, atmospheric pressure, and either the dew point temperature or the relative humidity.
- Introduction of the water temperature as a parameter affecting the wet bulb temperature leads to a more realistic result than published in the current ASHRAE Handbook.
- An accuracy of 0.01 K is typically reached in 2 to 5 iteration steps.
- The discrepancy between this method and the AHH reference software is usually no more than several hundredths of a degree.

6 Nomenclature

Quantity	Description	Unit
h	Specific enthalpy of moist air	kJ/kg
h_s^*	Specific enthalpy at the wet bulb	kJ/kg
h_w	Specific enthalpy of water	kJ/kg
p	Atmospheric pressure	hPa
p_w	vapor pressure of unsaturated air	Pa
p_{ws}	Dew point saturation vapor pressure	Pa
p_{ws}^*	Wet bulb saturation vapor pressure	Pa
T	Dry bulb temperature	°C
T_k	Dry bulb temperature (absolute)	K
T^*	Wet bulb temperature	°C
T_k^*	Wet bulb temperature (absolute)	K
T_{dp}	Dew point temperature	°C
$T_{dp,k}$	Dew point temperature (absolute)	K
W	Absolute humidity	kg/kg
W_s^*	Absolute humidity at the wet bulb	kg/kg
W_{dp}	Absolute humidity at the dew point	kg/kg
ϕ	Relative humidity	%

References

[1] American Society of Heating, Refrigerating and Air-Conditioning Engineers, 2009 ASHRAE Handbook-Fundamentals (SI) (2009).

[2] Cert.-Eng. Marin Zeller TU, VDI, Air Humid Handling 7.1, http://www.zcs.ch/en/software-products/ahh-software.html (2000).

[3] California Energy Commission, 2010 Appliance Efficiency Regulations, CEC-400-2010-012 (2010).

[4] American Society of Heating, Refrigerating and Air-Conditioning Engineers, ANSI/ASHRAE Standard 133-2008, Method of Testing Direct Evaporative Air Coolers (2008).

[5] American Society of Heating, Refrigerating and Air-Conditioning Engineers, ANSI/ASHRAE Standard 143-2008, Method of Test for Rating Indirect Evaporative Coolers (2007).